Insta 2011-2012 1ere session

$\Sigma = 23$ 1cm
majoration borne à 20

$\times 1/2$ si principe OK mais calcul faux

C1

Partie 1

1) $\boxed{M(x) = H(l-x) + (\delta h - v(x))F}$ (1)

2) $EI v'' = M(x) \Rightarrow EI v'' + Fv = F\delta h + H(l-x)$

$$\Leftrightarrow \quad v'' + \frac{F}{EI} v = \frac{F\delta h}{EI} + \frac{H}{EI}(l-x)$$

en posant $\omega^2 = F/EI \rightarrow v'' + \omega^2 v = \omega^2 \delta h + \alpha\omega^2(l-x)$

et $H = \alpha F \quad \rightarrow \boxed{v'' + \omega^2 v = \omega^2(\delta h + \alpha(l-x))}$ (1)

SG ESSM $\boxed{v_g(x) = A\cos\omega x + B\sin\omega x}$

3) . forme analogue au 2nd membre $\rightarrow$ linéaire $v_p(x) = cx + d$

. $v_p''(x) = 0 \rightarrow \omega^2 v_p = \omega^2(\delta h + \alpha(l-x))$ (1)

$\rightarrow \boxed{v_p = \delta h + \alpha(l-x)}$

4) $\rightarrow \boxed{v(x) = A\cos\omega x + B\sin\omega x + \delta h + \alpha(l-x)}$ (1)

5) CL

(4)

$$\begin{array}{|l|l}
v(0) = 0 & = A + \delta h + \alpha l \quad (1) \quad 1p \\
v'(0) = 0 & = -A\omega\sin 0 + B\omega\cos 0 - \alpha = B\omega - \alpha \quad (2) \quad 1p \\
v(l) = \delta h & = A\cos\omega l + B\sin\omega l + \delta h \quad (3) \quad 1p
\end{array}$$

$(2) \Rightarrow B = \frac{\alpha}{\omega} \qquad (1) \Rightarrow A = -(\delta h + \alpha l)$

$(3) \Rightarrow -(\delta h + \alpha l)\cos\omega l + \frac{\alpha}{\omega}\sin\omega l + \cancel{\delta h} = \cancel{\delta h}$

$$\Rightarrow -\delta h\cos\omega l = \alpha l\cos\omega l - \frac{\alpha}{\omega}\sin\omega l$$

$$\Rightarrow \delta h = \frac{\alpha\left(\frac{\sin\omega l}{\omega} - l\cos\omega l\right)}{\cos\omega l} = \alpha\left(\frac{\operatorname{tg}\omega l}{\omega} - l\right)$$

$$\boxed{\delta h = \frac{Hl}{EI\omega^2}\left(\frac{\operatorname{tg}\omega l}{\omega l} - 1\right)} \longleftarrow \boxed{\delta h = \alpha l\left(\frac{\operatorname{tg}\omega l}{\omega l} - 1\right)} \quad 1p$$

6) $\delta h \to \infty$ si $tg\, \omega \ell \to \infty$

$$\to \omega \ell = \frac{\pi}{2} \Rightarrow \boxed{\omega = \frac{\pi}{2\ell}}$$

(1)

$$\omega^2 = \frac{\pi^2}{(2\ell)^2} = \frac{F_{cr}}{EI}$$

$$\to \boxed{F_{cr} = \frac{\pi^2 EI}{(2\ell)^2}}$$

Remarque : F_{cr} ne dépend pas de H.

7) $$tg\, x = \frac{\sin x}{\cos x} \approx \frac{x - \frac{x^3}{6} + \cdots}{1 - \frac{x^2}{2} + \cdots} = \frac{x\left(1 - \frac{x^2}{6}\right)}{\left(1 - \frac{x^2}{2}\right)} \simeq \frac{x(6 - x^2)}{6} \times \frac{2}{(2 - x^2)}$$

$$\Rightarrow \frac{tg\, x}{x} = \frac{6 - x^2}{2 - x^2} \times \frac{2}{6}$$

(2)

$$\Rightarrow \frac{tg\, x}{x} - 1 = \frac{x^2}{3\left(1 - \frac{x^2}{2}\right)} \simeq \frac{x^2}{3}$$

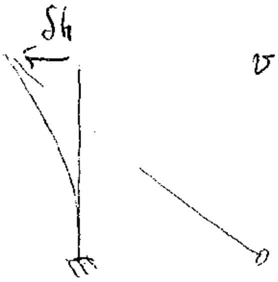
$$\Rightarrow \delta h = \frac{H \ell}{EI \omega^2} \times \frac{(\omega \ell)^2}{3} = \frac{H \omega^2 \ell^2 \ell}{EI \cancel{\omega^2}\, 3} = \boxed{\frac{H \ell^3}{3EI} \simeq \delta h}$$

Partie 2 :

8) 

$$F_{cr} = \frac{\pi^2 EI}{\ell_f^2} \quad \text{avec} \quad \ell_f = \frac{\ell}{\sqrt{2}}$$

(1)

9) 

$v = \delta h \left(\frac{x}{\ell}\right)^2$ est une parabole de sommet en $x = 0$

elle vérifie les conditions cinématique

(1)

$$\begin{cases} v(0) = 0 \\ v'(0) = 0 \\ v(\ell) = \delta h \end{cases}$$

10) $W_{fl} = \frac{1}{2}\int_0^{\ell} EI\, v''^2\, dx$

$$v = \delta h \left(\frac{x}{\ell}\right)^2$$

$$v' = \frac{\delta h}{\ell^2}\, 2x$$

$$v'' = \frac{2\delta h}{\ell^2} = cte \Rightarrow W_{fl} = \frac{1}{2}\left(\frac{2\delta h}{\ell^2}\right)^2 EI \underbrace{\int_0^{\ell} dx}_{\ell}$$

1,5

$$\Rightarrow \boxed{W_{fl} = 2\left(\frac{\delta h}{\ell^2}\right)^2 EI\,\ell} = 2\,\delta h^2 \frac{EI}{\ell^3}$$

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11) la force due au vent se deplace de δh → $\boxed{\delta h . H}$ 1

~~ne dérive pas d'1 potentiel~~ → 0

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12) $\delta v = \frac{1}{2}\int_0^{\ell} v'^2\, dx$

$$v' = \frac{2\delta h}{\ell^2}\, x \rightarrow v'^2 = \frac{4\delta h^2}{\ell^4}\, x^2$$

$$\frac{1}{2}\int_0^{\ell} v'^2\, dx = \frac{2\delta h^2}{\ell^4}\underbrace{\int_0^{\ell} x^2\, dx}$$

1,5

$$\left[\frac{x^3}{3}\right]_0^{\ell} = \frac{\ell^3}{3}$$

$$\Rightarrow \delta v = \frac{2\delta h^2}{\ell^4} \times \frac{\ell^3}{3} = \boxed{\frac{2\,\delta h^2}{3\ell} = \delta v}$$

$$W_e = |-F\delta v| = \boxed{\frac{2}{3}\frac{\delta h^2}{\ell} F = W_F}$$

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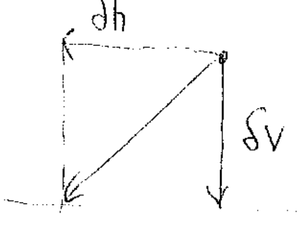
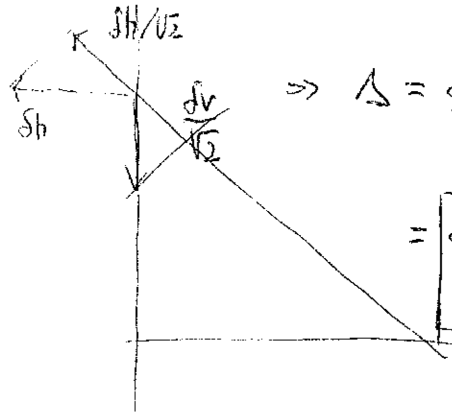
13) allongement Δ du câble dépend du déplacement du point B





$$\Rightarrow \Delta = \frac{\delta h}{\sqrt{2}} - \frac{\delta v}{\sqrt{2}}$$

2

$$= \boxed{\frac{\delta h}{\sqrt{2}} - \frac{2}{3\sqrt{2}}\frac{\delta h^2}{\ell} = \Delta} \approx \boxed{\frac{\delta h}{\sqrt{2}} \text{ si } \delta h \text{ petit}} \;*$$

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14) Energie elastique dans le câble

$$W = \frac{1}{2}\int_0^\ell \frac{N^2}{EA}\,dx = \frac{1}{2}\int_0^\ell EA\,\varepsilon^2\,dx$$

2

$$\frac{N}{EA} = \varepsilon = \frac{\Delta}{L_c}$$

L_c longueur du câble $= \ell\sqrt{2}$

$$\varepsilon = \frac{\Delta}{\ell\sqrt{2}} = \frac{\delta h}{2\ell} - \frac{2}{3\times 2}\frac{\delta h^2}{\ell^2} = \frac{1}{2}\frac{\delta h}{\ell} - \frac{1}{3}\left(\frac{\delta h}{\ell}\right)^2$$

$$\Rightarrow \varepsilon^2 = \left(\frac{\delta h}{\ell}\right)^2\left(\frac{1}{2} - \frac{1}{3}\frac{\delta h}{\ell}\right)^2$$

$$\Rightarrow \boxed{W_{câble} = \frac{1}{2}EA\ell\left(\frac{\delta h}{\ell}\right)^2\left(\frac{1}{2} - \frac{1}{3}\frac{\delta h}{\ell}\right)^2} \approx \frac{1}{2}\frac{EA}{\ell}\frac{\delta h^2}{4} \approx \frac{EA\delta h^2}{8\ell}$$

si δh petit

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15) le mât flambe lorsque F est tel que $\delta h\ \forall$

2

et $W_{ext} = W_{int}$

$$|\delta h\, H| + |\delta v\, F| = W_{flexion\ mât} + W_{câble}$$

*

$$\delta h\, H + \frac{2\delta h^2}{3\ell}F = \frac{2EI}{\ell^3}\delta h^2 + \frac{1}{2}\frac{EA\ell}{\ell^2}\delta h^2\left(\frac{1}{2} - \cancel{\frac{1}{3}\frac{\delta h}{\ell}}^{\,0}\right)^2$$

$$\Rightarrow \delta h \to \infty \text{ si } \boxed{F \approx \frac{3EI}{\ell^2} + \frac{3EA}{16}}$$